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**FRACTURE & DYNAMICS
PAPER NO. 62**

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SOME ASPECTS OF FORMATION OF CRACKS IN FRC WITH MAIN REINFORCEMENT

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Abstract

In this paper the response of fibre reinforced concrete with main reinforcement in pure tension is considered. Test results are presented showing three distinct regimes: a regime of linear elasticity, a regime of yielding at approximately constant stress, and finally, a regime of strain hardening. A simple model of the response of a tension member with main reinforcement and a partially opened crack is presented, and the influence of different shapes of the crack opening relation is studied. The case of a parabolic crack opening relation defines a brittleness number that describes the transition from discrete cracking to multiple cracking. It is shown, that if the crack opening relation is assumed to consist of a brittle contribution from the cement paste, and a more ductile contribution from the pull-out of the fibres, a plastic regime will be present in the tensile response. The fracture process is described from the uncracked state and formation of the first crack til the final stage where a large number of cracks have developed and the failure load is reached. The model is compared with experiments, and reasonable agreement is achieved.

1. Introduction

During the last decade, very strong concrete materials have been developed. Using low water-to-cement ratios, and densifying the material using small particles like microsilica - the so-called DSP concept - it is possible to make concretes with a compressive strength around 150 MPa.

The main problem using these new materials is their brittleness, i.e. their limited resistance to crack formation and crack extension. Thus, for structures built with these

new high strength concretes, cracks might be a more serious problem than for normal strength concrete. In some cases, structures built with these materials might develop cracks at unexpected low stress levels, and crack widths in these types of structures might be substantially larger, leading to larger sensitivity to environmental influences.

It has been shown e.g. Bache [1], that it is possible to use the new high strength materials in such a way, that the tendency to crack growth is decreased. In fact, by using high volume fractions of fibres combined with traditional reinforcement it is possible to obtain a ductility and a resistance to crack growth that is much larger than for normal strength concrete.

In practice, however, situations might arise where the use of a high strength concrete is needed, but where it is not the objective to use high volume fibre fractions, but just to add enough fibres to adjust the ductility of the high strength concrete, so that crack development and crack width might be controlled.

In this paper it is shown that if moderate fibre fractions are used, the tensile response of the material reinforced with main reinforcement might be dominated by initial yielding caused by formation of discrete cracks at a relatively low stress level. The plastic strains developed at this stage in the fracture process, might be large compared to elastic strains and plastic strains developed during later strain hardening.

In the literature many models have been proposed describing the cracking and the tensile response of concrete members with main reinforcement [5-10]. In most cases however, the models do not take into account the shape of the crack opening relation, or the models do not include debonding between reinforcement and matrix material. Some models include both effects, e.g. Stang and Aarre [5], but in this case, the model is relatively complicated taking into account other effects such as elastic shear in the concrete and influence of the Poisson ratio.

The idea of the work presented here, is to formulate the simplest possible model taking into account debonding as well as the shape of the crack opening relation. The model follows the basic ideas presented in the classical paper by Aveston, Cooper and Kelly, [10], but as an extension of the Aveston model, a partially opened crack is considered. The model provides a simple basis for analysing the uniaxial response of a fibre reinforced specimen with main reinforcement. The model gives the solution to the strain contribution due to partially opening cracks, and it provides a direct way of analysing the influence of the crack opening relation on the crack development process.

For the case where the crack opening relation is parabolic, the model leads to a brittleness number describing the transition from multiple cracking to formation of larger discrete cracks. For cases where the crack opening relation gets a large contribution from the DSP sub-matrix active only for very small crack openings, the model predicts the above mentioned initial yielding. A simple analytical expression is given for plastic strain developing during initial yielding.

The model is compared to experimental results, and it appears to be able to predict observed behaviour qualitatively correct.

2. Test results

The test results presented here, are a part of an extensive programme carried out in co-operation between Michigan University, Ann Arbor, and Aalborg University, Denmark, see Al-Shannaq [4].

The tension test specimens presented here were made of a fibre reinforced matrix reinforced with threaded bars. The matrix material was a relatively brittle cement-based sub-matrix reinforced with 6 mm steel fibres, fibre volume fractions $V_f = 3\%$ and $V_f = 6\%$. The main reinforcement was varied over the reinforcement ratios $\varphi = 1\%$, $\varphi = 3\%$, and $\varphi = 5\%$, 1 % corresponding to 2 bars with an outer diameter 3 mm, 3 % corresponding to 3 bars with an outer diameter 4 mm, and finally 5 % corresponding to 5 bars with an outer diameter 4 mm.

The sub-matrix was a brittle, high strength cement mortar, a so-called DSP material (densified with small particles) made of white Portland cement, microsilica, superplasticizer, and fine quartz sand aggregates, Bache [1], Al-Shannaq [4].

It is well known, Bache [1], Tjiptobro & Hansen [2], Hansen et al. [3], that the behaviour of a relatively brittle material such as DSP can be substantially modified by adding discrete fibres in a suitable volume fraction $V_f > 2\%$. The main effect of adding fibres is an improved ductility. The tensile strength is usually not affected much.

The fibres used in this investigation were brass coated steel fibres with a diameter of 0.15 mm. The tensile strength of the fibres was 2950 MPa. The properties of the interface between fibres and the DPS sub-matrix have been studied in detail in Al-Shannaq [4]. Tests showed debonding energies in the range 10 - 50 N/m, and friction stresses during debonding in the range 4 - 6 MPa.

The bars used as reinforcement had metric threads, the effective area of the bars $A_{ef} = 0.695A_{nom}$, where the nominal area is determined from the outer diameter. The exact reinforcement ratios based on the effective areas were 1.09 %, 2.91 % and 4.85 %. The failure stress based on the nominal area was 480 MPa for 3 mm bars, and 450 MPa for 4 mm bars, and the nominal Young's modulus (based on the outer diameter) was 135000 MPa.

The tensile specimens were rectangular plates of size $300 \times 75 \times 12$ mm, figure 1.a, the cross-section having the area $A = 12 \times 75 = 900 \text{ mm}^2$. The specimens were tested in a servo controlled 40 kN load frame in displacement control, figure 1.b. The strain was measured with strain gauges and with two clip gauges with a measurement length of 180 mm.

Some typical test results showing stress response to the total strain estimated from the clip gauge measurements are shown in figure 2. Figure 2.b shows the response of a DSP material with 6 % fibres reinforced with 1 %, 3 %, and 5 % main reinforcement. As it appears, the response is close to the response of a material with linear strain hardening,

and there is a smooth transition from the elastic regime to the strain hardening regime. Results from similar tests on a DSP material with 3 % fibres are shown in figure 2.a. As it appears from the results, in this case, the behaviour is distinctly different. The transition from the elastic regime to the strain hardening regime is no longer smooth as for the case with 6 % fibres, the transition is governed by initial yielding, i.e. before the strain hardening regime starts, an approximately horizontal regime appears in the response. As it appears, this initial yielding is present for all reinforcement ratios, but the associated plastic strain decreases with increasing reinforcement ratio.

3. Model of Partially Opened Crack

A specimen with cross-section area A subjected to uniaxial tension is considered. The specimen consists of a matrix reinforced with m continuous reinforcement bars with radius r . Thus the reinforcement ratio is $\varphi = m\pi r^2/A$. Since the idea of this model is to consider crack development in the case of a relatively tough matrix, the matrix itself should be considered as a cement based material (the sub-matrix) reinforced with fibres small enough to allow the fibre reinforced material to be treated as a continuum. The Young's moduli of the matrix and the reinforcement are E_m and E_r , respectively, defining the ratio $n = E_r/E_m$.

For a given uniform strain ϵ , the elastic stresses in the reinforcement and the matrix are easily calculated defining the composite stress $\sigma = (E_r\varphi + (1 - \varphi)E_m)\epsilon$ and the composite Young's modulus $E = \varphi E_r + (1 - \varphi)E_m$. Now, assuming that the matrix is the weak phase with tension strength f_t , the critical composite stress (the external stress corresponding to beginning failure of the matrix) is found to

$$\sigma_c = ((n - 1)\varphi + 1)f_t \quad (1)$$

At this state, the reinforcement stress is $\sigma_r = n f_t$, thus the force in the reinforcement is $F_c = n f_t \varphi A$.

Now, let us assume, that a crack starts opening uniformly over the cross-section, and let the matrix material have a certain crack opening response curve, figure 4. The crack is assumed to have the crack width w corresponding to the reduction s_m of the matrix stress. Thus, the matrix stress in the crack is $\sigma_m = f_t - s_m$, and requiring equilibrium, the corresponding force in the reinforcement is obtained

$$F = n f_t \varphi A + s_m (1 - \varphi) A \quad (2)$$

The state of stress around the crack changes when the crack opens. A simple model of the stress re-distribution is given by assuming plane deformation and that the reinforcement is partially debonding over a zone with depth a , the softening shear stress in the fibre matrix interface being constant equal to τ_f . Now consider a piece of reinforcement around

the crack. At the crack, the force is given by eq. (2), and at the depth a the force is given by $n f_t \varphi A$ as explained above. Assuming a constant shear stress τ_f at the interface, the length a of the debonding zone is obtained by equilibrium

$$a = \frac{(1 - \varphi)r}{2\tau_f\varphi} s_m \quad (3)$$

Assuming plane deformation in the reinforcement as well as in the matrix material, the stress disturbance introduced by the crack will be a linear deviation from the initially constant strain field. Thus, the unknown crack opening is easily obtained in terms of the stress change s_m by integrating the difference of strain in the reinforcement and the matrix

$$\begin{aligned} w &= 2 \int_0^a (\epsilon_r - \epsilon_m) dx \\ &= \left(\frac{1 - \varphi}{\varphi} \frac{1}{E_r} + \frac{1}{E_m} \right) a s_m \end{aligned} \quad (4)$$

and using eq. (3) the following expression is obtained for the crack opening

$$w = \frac{(1 - \varphi + n\varphi)(1 - \varphi)r}{2\tau_f E_r \varphi^2} s_m^2 = \frac{E(1 - \varphi)r}{2\tau_f E_r E_m \varphi^2} s_m^2 \quad (5)$$

The solution is illustrated in figure 4 showing the relation between the matrix stress and the crack opening for the solution given by eq. (5) and the material resistance given by the crack opening relation for the matrix material. The model solution is a horizontal parabola centred at $\sigma_m = f_t$ at the vertical axis. Two cases are shown, one for a relatively weak reinforcement, and one for a strong reinforcement. For both cases it is seen that since the derivative of the model solution is approaching infinity for $w \rightarrow 0$, close to the vertical axis, the resistance is larger than the model solution. Thus, if no initial cracks are present, the state $\sigma_m = f_t$, $w = 0$ is stable. However, if the crack has opened to a certain degree, so that the resistance becomes less than the model solution, the state is unstable, and the crack will open uncontrolled. Further, it is noticed, that for the case with a strong reinforcement, the crack must be forced through a larger set of stable states before reaching the unstable state. Thus, increasing the amount of reinforcement will help stabilizing existing cracks as it would be expected. As it appears, the same effect is achieved by decreasing the radius r of the reinforcement or by increasing the friction stress τ_f .

4. Some Theoretical Solutions

The simple model proposed in the preceding sections provides a basis for definition of a brittleness number B_r for the reinforced material. Assume that the crack opening relation for the matrix material is parabolic like the model solution, figure 5.a. In this case, if and only if the area G_r under the model curve is less than the fracture energy G_f of the matrix, all crack opening states will be stable. Thus, the stability criterion is $G_r < G_f$ defining the brittleness number $B_r = G_r/G_f$ and the stability criterion $B_r < 1$. The defined brittleness number is easily obtained

$$B_r = \frac{1}{6} \frac{E(1-\varphi)r f_t^3}{\tau_f E_r E_m G_f \varphi^2} = \frac{1}{6} \frac{E(1-\varphi)f_t}{\tau_f E_r \varphi^2} B_0 \quad (6)$$

where $B_0 = f_t^2 r / (E_m G_f)$ is the traditional brittleness number for homogenous materials, [1] where the reinforcement radius r is used as the characteristic length.

The value of the brittleness number might be taken as a guideline for the type of crack formation in the reinforced material. If B_r is well below one, stable cracks are expected resulting in a well distributed crack pattern. On the other hand, if B_r is well above one, unstable cracks are expected to form resulting in formation of larger discrete cracks.

Sometimes the shape of the crack opening is far from being parabolic, and an analysis based on the brittleness number defined above might be misleading. An important case is when the matrix is lightly or moderately fibre reinforced. In this case, the first part of the crack opening relation is governed by crack formation in the cement mortar resulting in a sharp peak at the beginning of the crack opening relation. Because of their brittle nature, it is difficult to measure these effects experimentally, but the tendency has been observed, see e.g. Li et al. [6]. For this case therefore, the crack opening relation is idealized as a delta spike at $w = 0$ followed by a constant level, figure 5.b.

Assuming a delta spike with height Δf_t , the elongation of the specimen due to formation of the crack is found by taking the contribution w_r to the crack opening from the reinforcement only

$$w_r = \frac{1-\varphi}{\varphi E_r} a \Delta f_t \quad (7)$$

The finite crack opening caused by the spike, will result in an initial plastic strain $\Delta \epsilon$ at the stress equal to the critical composite stress σ_c , figure 6. Assuming that the cracks form with the distance $2a$, where a is given by eq. (3), the initial plastic strain is found as

$$\Delta \epsilon = \frac{w_r}{2a} = \frac{1-\varphi}{\varphi E_r} \Delta f_t \quad (7)$$

Theoretically, the distance between the cracks is in the range from a to $2a$. Thus, the expression given above underestimates the plastic strain.

5. Model Validation

The simulated responses using the model presented in the preceding sections are shown in figure 7.

The crack opening relations were estimated from the test results fitting a function of the form $\sigma = f_t / (1 - (w/w_0)^p)$ as proposed by Stang and Aarre [5]. However, since the crack opening displacement relations were measured on notched specimens, and since cracks will form at the weakest points along the specimen, the measured crack opening stresses should be reduced. Thus, the estimated crack opening relations were reduced by a constant factor. Further, since the fibre contribution for the case of 3 % fibres is believed to be smaller than the contribution from the DSP sub-matrix for very small crack openings, in this case, the crack opening relation was supplied with a delta spike at the beginning. Thus, for 3 % fibre reinforcement, the values $f_t = 10$ MPa, $\Delta f_t = 3.2$ MPa were used, and for 6 % fibre reinforcement the values $f_t = 11$ MPa, $\Delta f_t = 0$ MPa were used. The crack opening relations used in the simulations are shown in figure 8. For the shear resistance the value $\tau_f = 10$ MPa was used.

The model was slightly modified to take into account higher stress levels than those considered in the presentation of the model. For higher stress levels, the solution presented in the preceding sections will introduce stresses in the material between the cracks that are larger than the tensile strength. This is acceptable to a certain extent. The first cracks will form in areas with low strength, thus, stresses away from the crack might be larger than the tensile stress of the material in the crack zone. Thus for composite stresses only moderately larger than the critical stress σ_c , the presented model might be an acceptable approximation. For larger stresses, however, the strain contribution due to partial failure of the material between the model cracks must be estimated. In this investigation, this contribution has been incorporated using a simple plasticity model for the matrix material between model cracks. For the matrix material between cracks a yield stress of $f_y = 11$ MPa was used for the 3 % fibre case, and $f_y = 12$ MPa for the 6 % fibre case.

As it appears from figure 7, the model is able to predict a response qualitative close to the response measured in the experiments. The model predicts the right tendencies concerning critical stress, initial yielding and strain hardening as well as final failure load. However, a closer examination shows some deviations. First, the model has a tendency to underestimate the strains. This is true for the initial plastic strain as well as the plastic strain developed during strain hardening. Partly this might be due to the simple assumption that the distance between cracks is twice the calculated debonding length. As mentioned earlier, this leads to an underestimation of the strain contribution due to cracking. Further, the final failure loads are overestimated. This is believed to be due to the simple model of the reinforcement response. In the model the reinforcement

was modelled as being linear elastic material up to the failure load. This is a rough approximation close to failure where yielding and necking might substantially decrease the reinforcement stiffness. If the stiffness is decreased in the model, this will lead to larger crack widths, and this again will lead to smaller matrix stresses tending to reduce the final failure load estimated by the model.

6. Conclusions

In this paper test results have been presented showing that there is a transition from a smooth behaviour when high fibre volume fractions are used together with main reinforcement to a behaviour with initial yielding - plastic strain development at approximately constant stress preceding the hardening regime - at moderate fibre contents.

A model has been presented that takes into account the shape of the crack opening relation as well as debonding of the reinforcement around the crack. The model is extremely simple, and in its most simple form, it gives a suitable approximation only for the material response around the point of first cracking. For higher stresses, the model must take into account partial failure of the material between cracks.

The presented model provides a direct way of analysing the influence of the shape of the crack opening relation. Two cases are considered. One case is when the crack opening relation is parabolic. For this case, a brittleness number is defined describing the transition from a response without initial yielding dominated only by multiple cracking and strain hardening (high fibre volume fractions) to a response with formation of larger discrete cracks (lower fibre volume fractions). The other case considered is when the initial part of the crack opening relation is dominated by the contribution from the sub-matrix. Knowing the strength contribution from the sub-matrix and the fibre reinforcement, a simple analytical solution provides an estimate of the plastic strains developed during initial yielding.

The model compares well with the experimental results and predicts the right tendencies concerning critical stress, initial yielding and strain hardening as well as final failure load. However, the model tends to underestimate plastic strains and overestimate the final failure load. These deviations might be explained by some of the simplifications introduced in the modelling.

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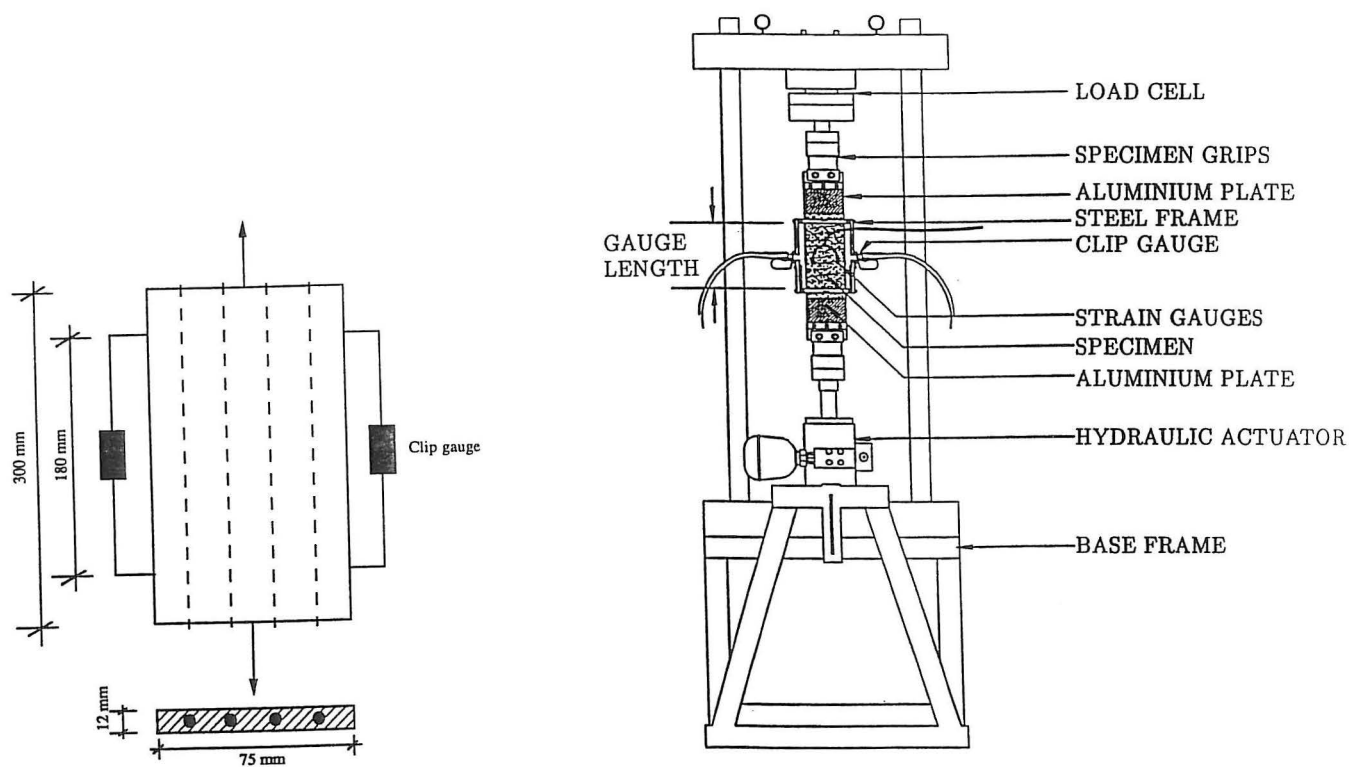


Figure 1. Specimen and test setup.

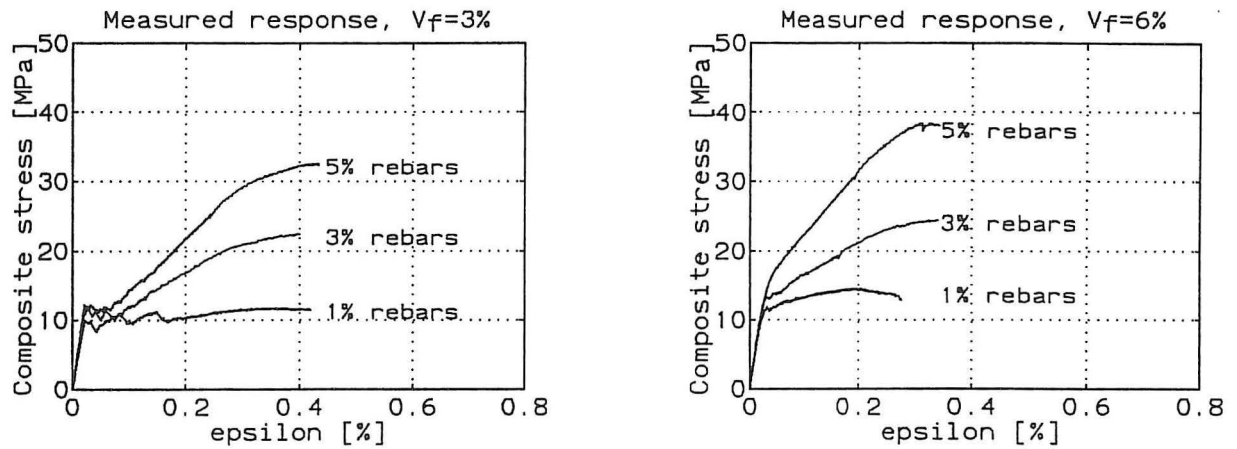


Figure 2. Measured responses in uniaxial tension. 2.a: response for 3 % fibre reinforcement, 2.b: response for 6 % fibre reinforcement.

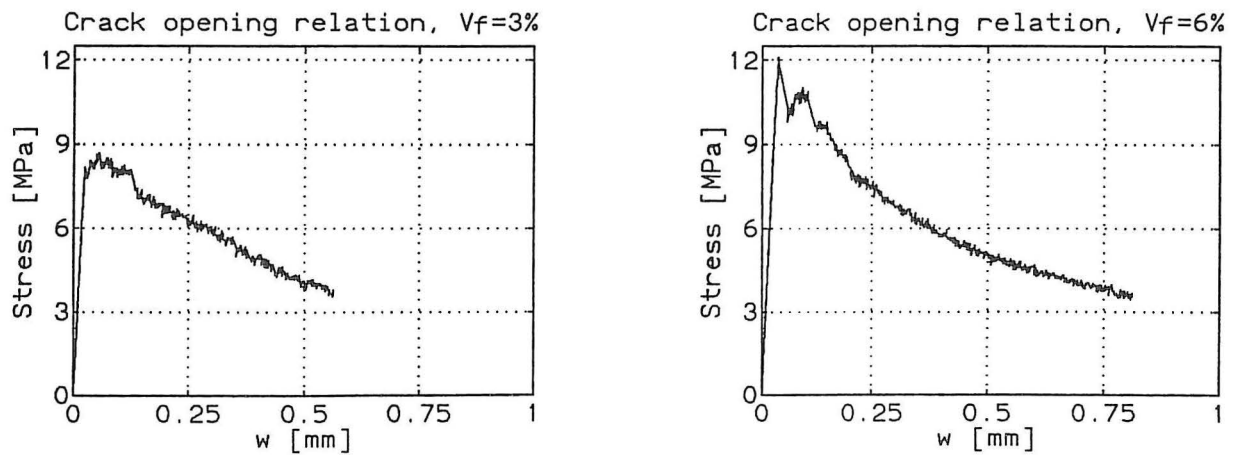


Figure 3. Measured responses for notched specimens without main reinforcement. 3.a: response for 3 % fibre reinforcement, 3.b: response for 6 % fibre reinforcement.

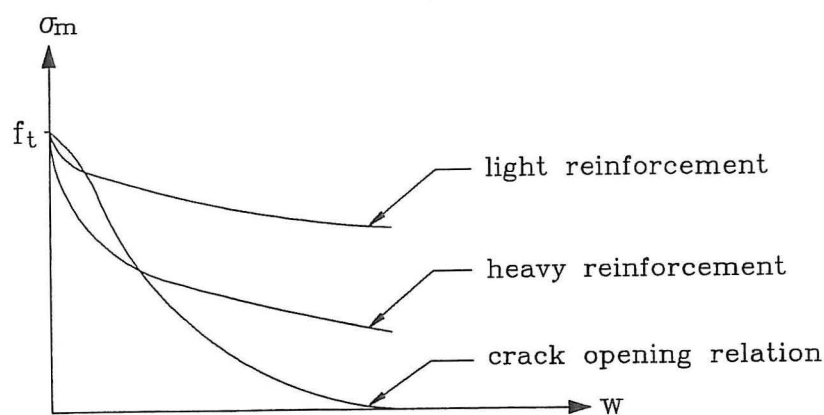


Figure 4. Model solutions for light and strong main reinforcement compared to material resistance, the crack opening relation.

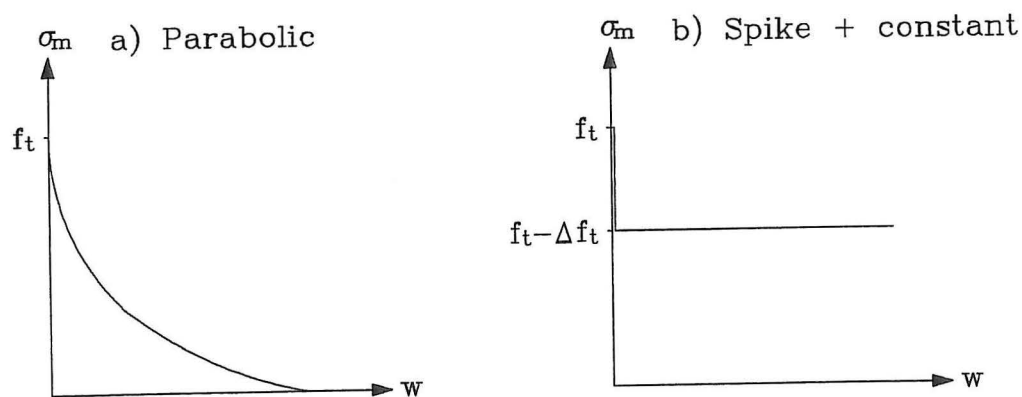


Figure 5. Idealised crack opening relations. 5.a: parabolic crack opening relation, 5.b: constant crack opening relation with an initial spike due to DSP contribution.

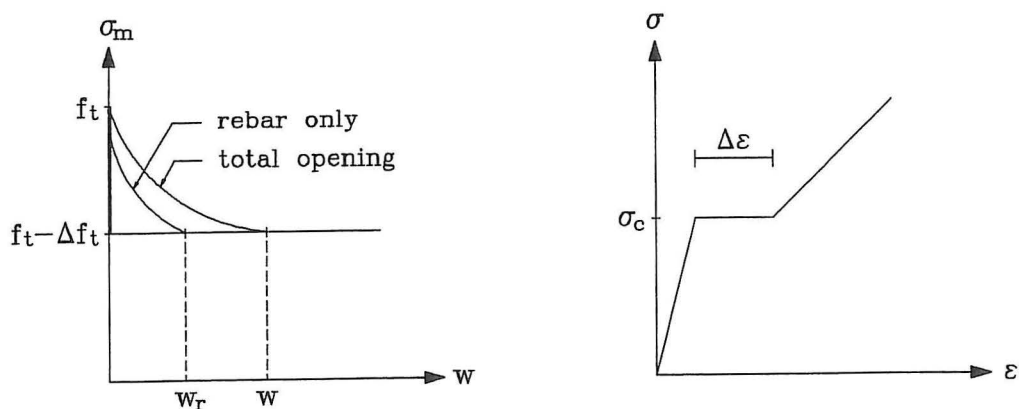


Figure 6. Estimating the plastic strain due to initial yielding. 6.a: estimating the elongation of specimen due to formation of cracks, 6.b: definition of the plastic strain due to initial yielding.

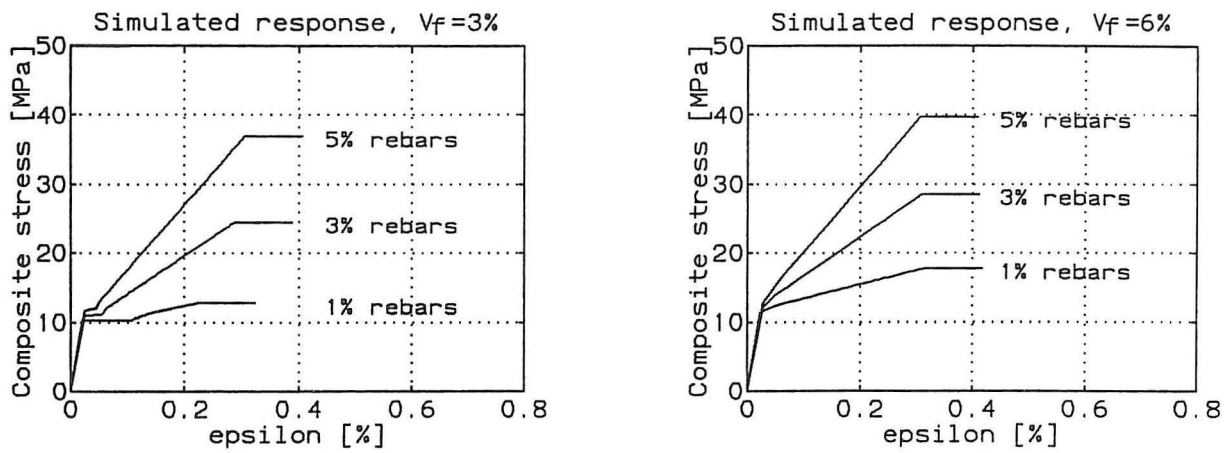


Figure 7. Response in uniaxial tension simulated by the presented model. 7.a: response for 3 % fibres, 7.b: response for 6 % fibres.

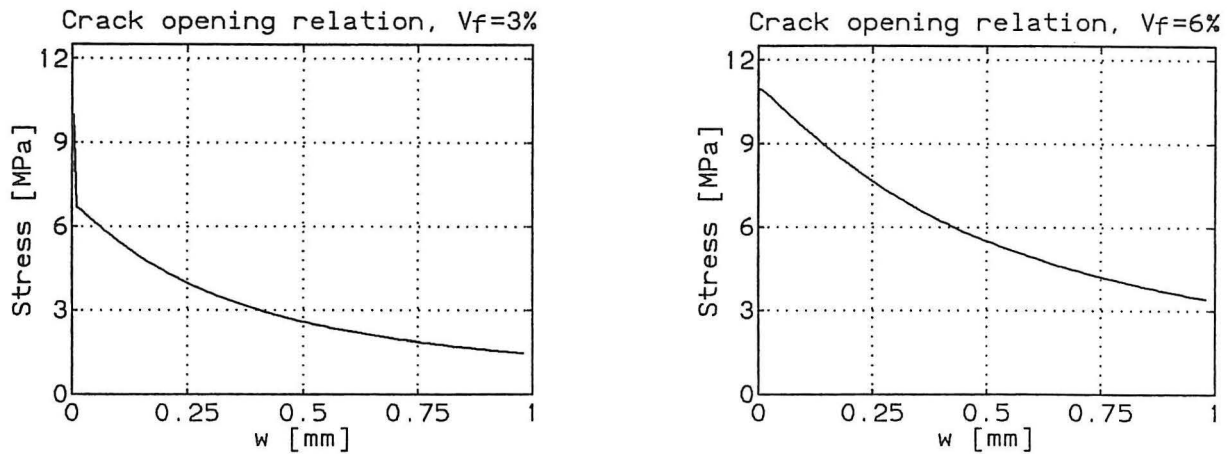


Figure 8. Crack opening relations used in the simulations. 8.a: crack opening r-elation for 3 % fibres, 8.b: crack opening relation for 6 % fibres.

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